

Introduction

This experiment studies the transfer of heat in a fin surrounded by air. The fin is an aluminum rod surrounded by air, with the bottom of the rod at either a constant or oscillating temperature. If there is a temperature difference between the rod and the surrounding air, or between different ends of the rod, then heat will flow from the hotter areas to the cooler ones. This experiment studies the transfer of heat within the rod and between the rod and its surroundings.

The three methods by which heat is transferred are conduction, convection, and radiation. When solving for the heat profile of a fin, we assumed that conduction—the process by which thermal energy is transferred between adjacent molecules due to a temperature gradient—was the only method of heat transfer within the fin. When heat is transferred by conduction, neighboring molecules vibrate against each other or transfer electrons to each other. Because the metallic bonds in metals have free electrons, metals, such as aluminum, are good conductors of heat.

Convection occurs when the movement of molecules transfers heat from one location to another. Conduction therefore increases as the fluid motion increases and does not occur in solids (which are not fluids). When the temperature of the air surrounding the metal rod is different than the temperature of the rod, heat will be transferred through the surrounding air by convection.

Radiation is the transfer of heat through electromagnetic waves, through empty space. All objects radiate heat at a rate proportional to their emissivity. Because the emissivity of aluminum is very low, we assume that the heat transferred by radiation through the rod is negligible.

This experiment studies the temperature profile of the rod under two different conditions. First, we held the bottom of the rod at a constant temperature and studied the steady-state heat transfer through and from the rod. Next, we applied a sinusoidal time-varying temperature to the bottom of the rod, and studied the transient heat transfer through and from the rod. By using dimensional analysis and analyzing our data, we were able to solve for the dimensionless constants of our experiment.

Setup

This experiment builds on the thermal control experiment that we completed in the previous lab. We modified the thermoelectric cooler (TEC) setup and the LabVIEW code from the “Thermal Control via LabVIEW” experiment slightly for this experiment:

First, we plugged one end of a cylindrical rod into the TEC and left the other end in the air. The rod is aluminum, with a 1.2 cm diameter. We also added five additional thermistors and placed them along the length of the rod. Each thermistor is held in place by a ring around the rod, and we used thermal glue to increase the thermal conduction between the rod and the thermistors. We also left the one thermistor at the bottom of the rod, touching the TEC. A block of insulation was placed around the TEC, so we did not use the data from the bottom thermistor in any of our calculations.

Next, we modified our LabVIEW code to include temperature data from all six thermistors. Temperature-versus-time data from all six thermistors is plotted on the same graph, and the data is automatically saved as a text file.

Lastly, we changed our LabVIEW program so that the base temperature can oscillate sinusoidal over time about a set temperature T_0 . The user can set T_0 , as well as the oscillation amplitude and period. If a constant base temperature is desired, the amplitude can be set to zero.

Our code (block diagram) and the user interface (front panel) are shown below, in Figures (1a) and (1b), respectively.

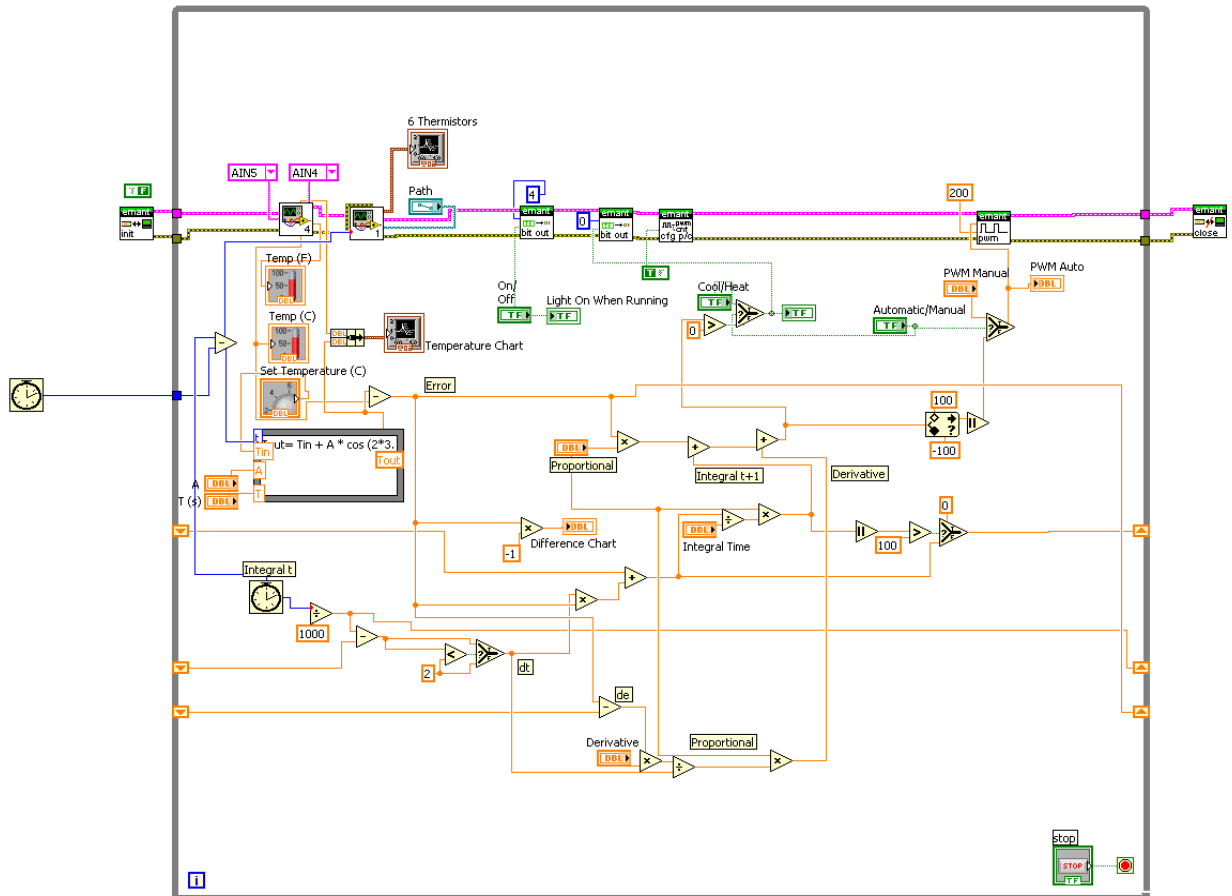


Figure (1a): LabVIEW code: Block Diagram

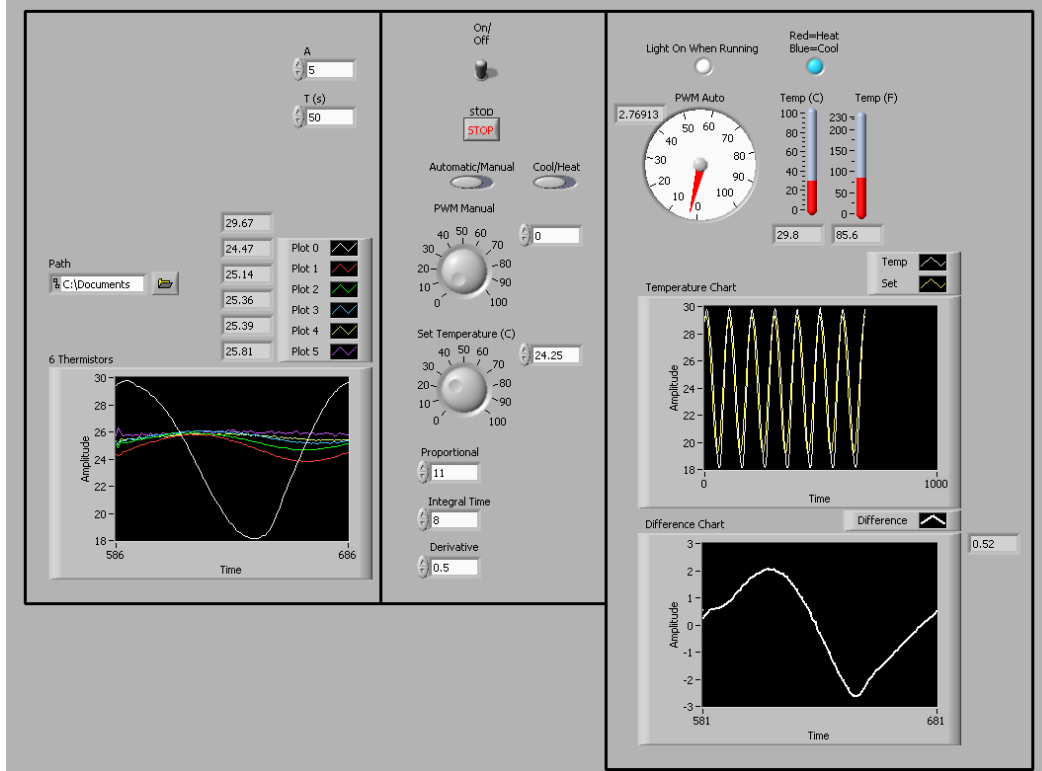


Figure (1b): LabVIEW program: Front Panel

Theory

This experiment studies the heat profile of a long cylinder whose base is set at a constant or oscillatory temperature while the rest of the rod is exposed only to room-temperature air. The heat is transferred along the axis of the rod by conduction and from the sides of the rod into the air by convection. Fourier's Law of Heat Conduction describes the convective heat transfer, while Newton's Law of Cooling describes the convective heat transfer. The First Law of Thermodynamics is a statement of conservation of energy. By combining these three laws, we are able to calculate the temperature profile along the rod.

First, I will discuss the First Law of Thermodynamics, which states that the increase of a closed system's internal energy is equal to the sum of the energy added by heating the system and the work done on the system by its surroundings. In other words, energy can change from one form to another, but it must always be conserved. In Equation (1), a statement of the First Law of Thermodynamics, W is the transfer rate of work done by the system on its surroundings, U is the internal energy of the system, t is time, and Q is the heat transfer rate.

$$(1) \frac{dU}{dt} = Q - W$$

Assuming that no work is done on the system:

$$(2) Q = \frac{dU}{dt} = mc \frac{dT}{dt}$$

The outermost parts of Equation (2) are a well-known thermodynamics equation, which relates the heat flow into or out of an object to the temperature change dT of the object. The mass m has a specific heat capacity c , a constant that depends on the material of the object.

Second, I will examine Fourier's Law, which states that when heat is transferred by conduction, the rate of heat transfer is proportional and opposite in sign to the temperature gradient. Equation (3) is Fourier's Law, where q is the heat flux, or the heat transfer rate per unit area; k is the proportionality constant; and ∇T is the temperature gradient. The negative sign says that heat will always flow from hotter temperatures to cooler temperatures.

$$(3) \mathbf{q} = -k \nabla T$$

If the heat transfer is one dimensional, Equation (3) is simplified into Equation (4), where x is the direction in which the heat is flowing.

$$(4) q = -k dT/dx$$

By definition of flux, the heat flux, q , can also be expressed in terms of the rate of heat transfer, Q . The relationship is $Q = q A$, where A is the surface area of the object.

Lastly, Newton's Law of Cooling describes convective heat transfer. Newton discovered that when a cooler fluid is flowing past a hotter object, the rate at which the object's temperature changes is proportional to the difference between the object's temperature and the fluid's temperature. From Equation (2), it is obvious that the rate at which an object's temperature changes is also proportional to rate of heat transfer. If the object's temperature is held constant over time (due to an energy source that heats the object steadily as it cools), then Newton's Law takes the form of Equation (5):

$$(5) q = h (T - T_{\infty})$$

In Equation (5), h is the constant of proportionality, called the heat transfer coefficient, and varies greatly for different situations. Because h varies so greatly, it is not truly a constant along the surface of the aluminum rod. For the purposes of this lab, we will assume that h is the average value of the heat transfer coefficient over the entire surface of cooling object. T_{∞} is the temperature of the fluid far away from the object, in our case room temperature.

As promised, the First Law of Thermodynamics, Fourier's Law of Heat Convection, and Newton's Law of Cooling by convection together lay the basis for the heat transfer equation for a fin. A fin is an extension of a surface. For example,

the aluminum rod with one end touching the TEC is one type of fin. As the fin's surface area increases, the convective heat transfer also increases.

As mentioned above, within the rod, conduction is the process by which heat is transferred. We will assume that the conduction within the rod occurs only in the x -direction, so that the temperature along the length of the rod is only a function of x and t , and not of y or z , the directions perpendicular to the length of the rod. We will first study the case of steady state conduction, when the TEC is set to a constant temperature. After, we will discuss the transient case, when the TEC is set to an oscillatory temperature.

Figure (2) shows the setup of the fin. T_0 is the temperature of the TEC and the base of the rod, T_∞ is room temperature, A is the cross sectional area of the rod, P is the perimeter of the rod, L is the length of the rod, and h_L is the heat transfer coefficient at the tip of the rod. For a rod with an insulated tip, h_L is negligible, and we will assume for the moment that our rod has an insulated tip (we will later assume that the rod is infinitely long).

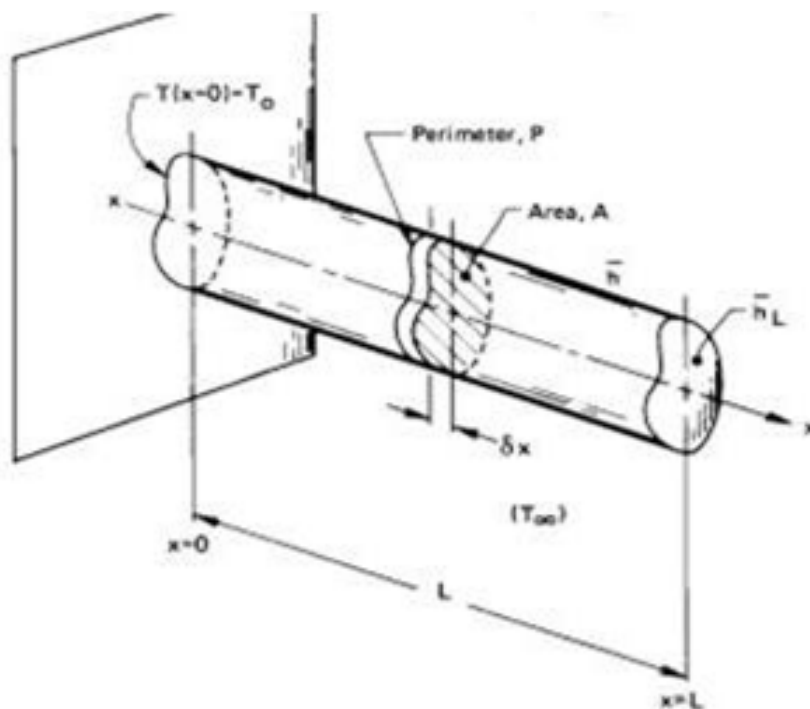


Figure (2): The Fin Setup

We can set up a differential equation for the heat flux along a section of the rod of length δx : Conservation of energy requires that the total energy along the δx is conserved. Thus, the difference of the heat flowing (conductively) through the rod at $(x + \delta x)$ and through the rod at x must be equal to the heat lost from the surface area of the rod (convectively) between x and $(x + \delta x)$. In other words:

$$(6) Q \text{ (conductively)}|_{x + \delta x} - Q \text{ (conductively)}|_x + \delta Q \text{ (convectively)}|_{\delta x} = 0$$

Using the fact that $Q = q A$ as well as Equations (4) and (5), we can rewrite Equation (6), where $(P \delta x)$ is the surface area of δx .

$$(7) -kA \frac{dT}{dx} \Big|_{x+\delta x} + kA \frac{dT}{dx} \Big|_x + h (P \delta x) (T - T_\infty) = 0$$

Equation (7) can be simplified further, into the form of Equation (8), where $q_l = \sqrt{(hP/kA)}$. Note that q_l is a dimensionless variable that is constant in a cylindrical rod, assuming that both h and k are constant throughout the rod. Equations (8) also give the boundary conditions for the problem.

$$(8) \frac{d^2(T - T_\infty)}{dx^2} = q_l^2 (T - T_\infty)^1$$

$$(T - T_\infty) \Big|_{x=0} = (T_0 - T_\infty)$$

$$\frac{d(T - T_\infty)}{dx} \Big|_{x=L} = 0$$

The general solution to the differential equation is $(T - T_\infty) = C_1 e^{+(q_l)x} + C_2 e^{-(q_l)x}$. Applying the boundary conditions, along with the assumption that the rod is infinitely long ($L \rightarrow \infty$) gives the solution:

$$(9) (T - T_\infty) = (T_0 - T_\infty) e^{-q_l x}$$

By taking data of $T(x)$ along the length of the rod, we are able to solve for the experimental value of q_l . We are also able to solve for the theoretical value of q_l if we know the constants of the setup. Lastly, using Equation (9b), we are able to solve for the rate of heat loss from the fin:

$$(9b) Q = (h P k A)^{1/2} (T_0 - T_\infty)$$

For the second part of this experiment, we studied set the base temperature of the rod to oscillate about room temperature. In other words, we set $T_0 = A_0 + \Delta T \cos(2\pi t/\tau + \varepsilon)$, where ΔT is the amplitude of the oscillation and τ is the period of oscillation.

Again, it is necessary to begin by setting up an expression for the rate of heat gain in an element of the rod δx . One again, the rate of heat transferred through the rod by conduction is given by $-kA \partial T(x, t)/\partial x$, and the rate of heat lost to surrounding through convection is $h(T - T_\infty) (P \delta x)^2$. An equation similar to Equation (7) can be written:

¹ $d^2(T)/dx^2 = d^2(T - T_\infty)/dx^2$, because T_∞ is not a function of x , so adding a constant into the differential term is allowed.

² The derivatives from above have been changed into partial derivatives, because the temperature is now a function of both x and t , while in the steady-state case above, it was only a function of x .

$$(10) \quad Ak \partial^2 T / \partial x^2 \delta x - h(T - T_\infty) (P \delta x) = Ac\rho \partial T / \partial t \delta x$$

The main difference between Equations (7) and (10) is the term on the right side of Equation (10). Because the temperature along the rod is no longer constant, $\partial T / \partial t$ is no longer equal to zero, and there is now a non-zero total rate of heat gain in the element δx , which is given by Equation (2). The mass m is now given in terms of its density ρ and volume $A \delta x$ ($m = \rho A \delta x$). Equation (10) can be rewritten in terms of a more concise form, where κ and ν are given by Equations (12) and (13):

$$(11) \quad \partial T / \partial t = \kappa \partial^2 T / \partial x^2 - \nu (T - T_\infty)$$

$$(12) \quad \kappa = k / (c\rho)$$

$$(13) \quad \nu = hP / (Ac\rho)$$

For simplicity, we will set $T = 0$ at T_∞ , and we will look for periodic solutions for $T(x, t)$ in the form Equation (14), where ω is the oscillation frequency. By substituting the assumed solution into Equation (11), we are left with the differential equation and boundary conditions given in Equation (15)³. The solution to Equation (15) is given in Equation (16).

$$(14) \quad T(x, t) = T(x) e^{in\omega t}$$

$$(15) \quad \partial^2 T / \partial x^2 = [(in\omega + \nu) / \kappa] T$$

$$T(x=0) = T_0$$

$$T(x=L) = T_\infty = 0$$

$$(16) \quad T(x) = T_0 \exp^{-[(in\omega + \nu) / \kappa]x}$$

The solution in Equation (16) can be rewritten as Equation (17), in terms of two new variables q_n and q_n' , as long as $-(q_n + iq_n')x = -[(in\omega + \nu) / \kappa]x$. q_n and q_n' are defined in Equations (18), and they satisfy the requirement above. Note also, that Equation (19) is an expression for the relationship between q_n , q_n' , and κ (but not ν).

$$(17) \quad T(x) = T_0 \exp[-(q_n + iq_n')x]$$

$$(18) \quad q_n = [[v + (v^2 + n^2\omega^2)^{1/2}] / (2\kappa)]^{1/2}$$

$$q_n' = [[-v + (v^2 + n^2\omega^2)^{1/2}] / (2\kappa)]^{1/2}$$

$$(19) \quad q_n q_n' = n\omega / (2\kappa)$$

³ The second boundary condition relies on the assumptions that the rod is infinitely long and that room temperature is the “zero point” on the temperature scale.

Now that we have solved the position dependence $T(x)$, we will look for a general solution for $T(x, t)$. Plugging Equation (16) into Equation (14), taking the real part of the solution, and writing $T(x, t)$ as the summation over the index n gives the general periodic solution, Equation (20). Because for our setup the base temperature is sinusoidal oscillating with time, meaning $T(x=0, t) = A_0 + A_1 \cos(\omega t + \varepsilon_1)$, Equation (20) is only expanded for the first two terms, giving Equation (21). Because only q_1 and q_1' appear in the solution, q_1 and q_1' will simply be referred to as q and q' from here on.

$$(20) \quad T(x, t) = \sum_{n=0}^{\infty} A_n \exp(-q_n x) \cos(n\omega t - q_n' x + \varepsilon_n)$$

$$(21) \quad T(x, t) = A_0 \exp(-q_1 x) + A_1 \exp(-q_1' x) \cos(\omega t - q_1' x + \varepsilon_1)$$

By taking data of $T(x, t)$ for different periods, we are able to solve for $q(\omega)$ and $q'(\omega)$. Using Equation (19), we can solve for κ , which we can in turn use to solve for ν . Of course, we are also able to solve for the theoretical values of κ and ν if we know the constants of our setup, using Equations (12) and (13). We can then use the theoretical values of κ and ν to solve for theoretical values for $q(\omega)$ and $q'(\omega)$. Comparing the theoretical values to our experimental results will indicate the accuracy of our experiment.

Procedure

After building the setup described above, we needed to first tune our PID (we got the best results with $k=11$, $T_i=8$, and $D=0.5$). We then began to take data. We first set the TEC at a constant temperature, and measured the displacement of the five thermistors along the length of the rod. We waited until the temperature was constant at each thermistor, and we recorded the steady-state temperature at each of the five x -positions along the rod.

Next, we set the base to a new temperature, and again waited until the temperature was steady before recording the data. We repeated this process for six different base temperatures (5°C, 10°C, 15°C, 40°C, 50°C, 60°C). We then moved the five thermistors to new locations, measured their positions, and repeated the process. With the $T(x)$ data for ten positions at six temperatures, we had all the data that we needed to calculate q .⁴

Next, we measured set the base temperature to oscillate with an amplitude of 5°C about room temperature. At each of the five thermistors, we recorded data of

⁴ Actually, we had more data than necessary. Because we would be fitting the data to a model with several parameters, we took more data than was strictly necessary for the calculations.

temperature versus time. We repeated this process for periods of 100s, 150s, 200s, 250s, and 300s. We then had enough data to calculate q and q' .⁵

The constants of our setup, that we used are used in the calculations below are as follows: the rod's diameter = 1.2 cm, $h = 10 \text{ W/m}^2\text{K}$, $\rho_{\text{aluminum}} = 2700 \text{ kg/m}^3$, $c = 900 \text{ J/K kg}$, and $k = 220 \text{ W/mK}$.

Experimental Results

We began by analyzing out steady-state data in order to solve for q_l , the dimensionless constant in Equation (9). As mentioned above, we took data of $T(x)$ along the length of the rod, while $T_0 = T(x=0)$ remained constant. In order to find our experimental value of q_l , we then fit our temperature-versus position data to a model in the form of the expected solution for $T(x)$, given by Equation (9). The model that we used is given in Equation (22).

$$(22) \quad \text{Model_}T(x) = T_\infty + (T_0 - T_\infty) \exp[-q_l (x + \delta)]$$

By setting, T_∞ , $(T_0 - T_\infty)$, q_l , and δ as variables, and using ten data points, we were able to find the experimental value of q_l . In Equation (22), δ is an added constant to increase the accuracy of our fitting, since we do not know exactly where the zero point is. Because we took six sets of data, corresponding to the six constant base temperatures, we were able to find six values for q_l . Figure (3) shows the graph of our fitted model, at a base temperature of 40°C. Figure (4) lists the experimental values of q_l that we found at each different base temperature. The average value was $5.57\text{m}^{-1} \pm 0.01\text{m}^{-1}$, with a standard deviation of 1.39m^{-1} .

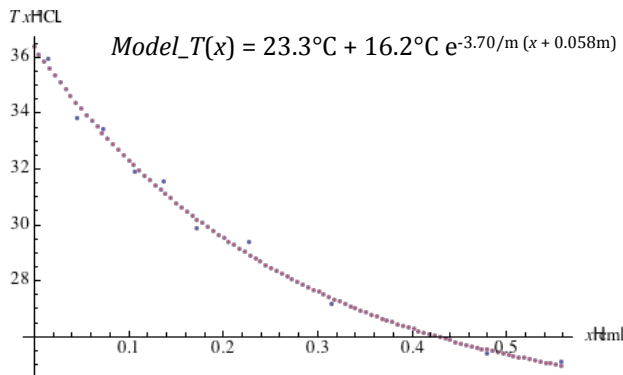


Figure (3): Model of $T(x)$ vs. x at $T_0 = 40^\circ\text{C}$

T_0 ($^\circ\text{C}$)	q_l (1/m)	% error
40	3.70	-5%
50	4.06	4%
60	5.74	48%
15	6.08	56%
10	6.87	77%
5	6.96	79%

Figure (4): Data of q_l (T_0)

We were also able to calculate the “true” value of q_l , by plugging the constants of our setup into the definition of q_l , defined above. Equation (23) shows that the theoretical value of q_l is 3.89m^{-1} . Figure (4) lists the percent error of each experimental q_l .

⁵ Again, we had more data than necessary. Because we would be fitting the data to a model with four parameters, we took five sets of data.

$$(23) \quad q_l = \sqrt{(hP/kA)} \\ = 3.89 \text{ m}^{-1}$$

It is clear from Figure (2) that the error increases as the difference between the temperature and room temperature increase, particularly as the $T_0 \ll T_\infty$. This is to be expected, because the approximations that we used in solving for the temperature profile of the rod become less and less accurate as the $(T_0 - T_\infty)$ increases. First of all, when the base temperature is very hot or cold, the tip or the rod was not exactly at room temperature, and the approximation that $T(x=L) = T_\infty$ is no longer entirely accurate. Secondly, the approximation that h is uniform is really only a rough approximation that does not hold for very large temperature differences. We expect that the most accurate experimental value for q_l is the one obtained when T_0 is nearest room temperature. Indeed, the values with the smallest error (5%) is $q_l = 3.70 \text{ m}^{-1} \pm 0.01 \text{ m}^{-1}$ (or $1/q_l = 0.270 \text{ m}$), obtained at 40°C .

To more accurately solve the for the heat profile of the rod, we would have to re-fit the data to the solution for a rod of finite length. However, because we had more data than necessary and only some of the data relied on inaccurate assumptions, we instead chose to use the value of q_l obtained at 40°C .

Other possible sources of error for this part of the experiment are that the data was taken on different days, when the room temperature varied slightly, and that we took the data at each temperature in two set (as discussed above, we only had five thermistors, so in order to get ten data points, we needed to reposition the thermistors). It is possible that we did not measure the position of the thermistors on the exact same scale, which would lead to errors in our data.

After solving the steady state heat profile in the rod, we analyzed our data for the oscillatory case. We took data of $T(x, t)$ for five different periods. We expected the data to be in the form of Equation (22), so we created a model for $T(x, t)$ based on Equation (22). The models is that we used are given in Equations (24). When all of Equations (24) are combined, they give a solution of exactly the same form as Equation (22), as they should⁶.

$$(24) \quad \text{Model_}T(x, t) = A_i + B_i \cos(\omega t + \varepsilon_i)$$

$$\text{Model_}A_i = (T_0 - T_\infty) \exp(-q_l x_i + \delta)$$

$$\text{Model_}B_i = \Delta T \exp(-q_l x)$$

$$\text{Model_}\varepsilon_i = \varepsilon_1 - q_l' x$$

⁶ The only difference between Equations (22) and (24) is that the models include fitting constants δ . Although the same symbol is used in various models, it is not constant from model to model. As discussed above, the only purpose of δ is to get rid of error due to us not knowing the zero point of the equations.

For each period, we fit our data to Equations (24) to solve for experimental values for $q(\omega)$ and $q'(\omega)$. As Equations (24) suggest, we fit our data in two steps. First, we used Model $T(x, t)$ to find A_i , B_i , ω , and ε_i for thermistor, for each period. We then took the five values that we obtained for A_i , B_i , and ε_i at each period, and fit them to Models A_i , B_i , and ε_i to find q and q' .

Model_{A_i} did not give us any new information. Because we set our TEC to oscillate about room temperature, $(T_0 - T_\infty)$ was equal to zero. However, this was not a problem, as we already calculated q_l from the steady-state case.

In contrast, Models B_i and ε_i allowed us to calculate values for q and q' for each period. For example, Figure (5a) shows the fitted model $T(x, t) = A_i + B_i \cos(\omega t + \varepsilon_i)$ for a period of 200s. The three, different-colored curves represent data taken at three different locations x_i . The curve with the largest amplitude is the nearest to the TEC. Figure (5a) also shows the increasing phase shift along the length of the rod. On the graph of Figure (5a), I have also included the fitted model for the blue curve, as an example. Figures (5b) and (5c) show the fitted curves the models for B_i and ε_i , respectively. Figures (5b) and (5c) also show the fitted models, which give experimental values for q and q' , respectively.

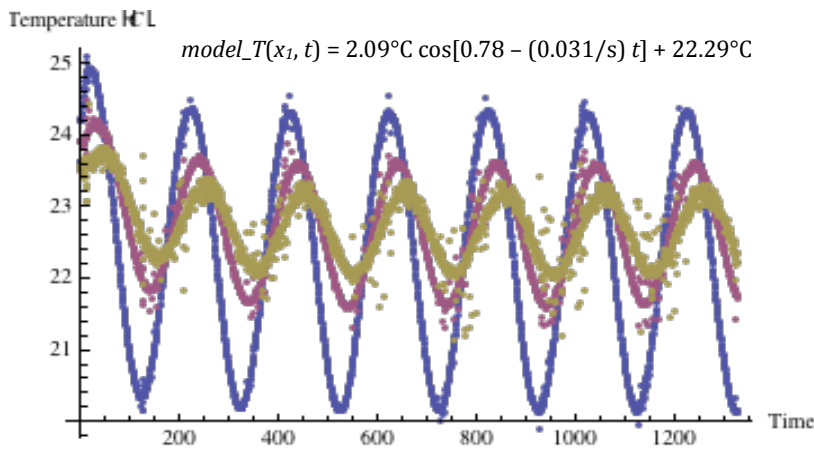
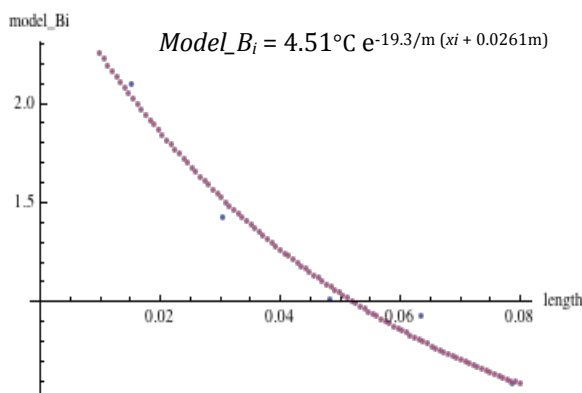
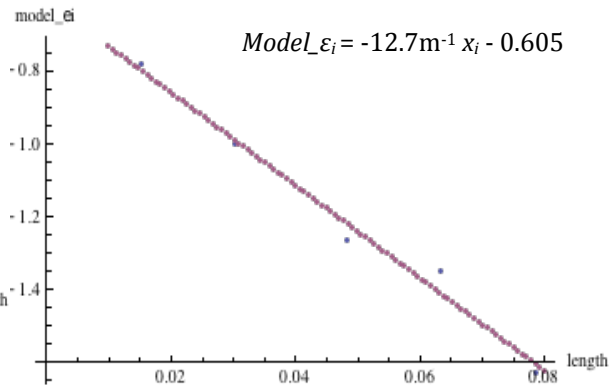


Figure (5a): Model $T(x, t)$ at period = 200s



Figure(5b): Model B_i at period =200s



Figure(5c): Model ε_i at period =200s

After fitting all of the data, to our models, we had five experimental values for $q(\omega)$ and $q'(\omega)$ ⁷. One way to determine the accuracy of our data is by calculating the theoretical values for q and q' for each frequency that we studied, and comparing the theoretical values to our experimental ones (Theoretical data is calculated using Equations (18)). This data is shown in Figure (6a), along with the percent error for each measured value. The errors range from 1% to 81%, suggesting that there was some error in our experiment. Possible sources of error are discussed below.

Period (s)	Measured q (m^{-1})	Theoretical q (m^{-1})	Error q	Measured q' (m^{-1})	Theoretical q' (m^{-1})	Error q'
100	26.4	18.8	40%	16.8	18.4	-9%
150	26.1	15.5	69%	7.4	15.0	-51%
200	19.3	13.5	43%	12.7	12.9	-1%
250	18.1	12.1	49%	13.5	11.5	18%
300	16.4	11.1	47%	18.8	10.4	81%

Figure (6a): Experimental and Theoretical data for q and q'

Figure (6b) summarizes the data in Figure(6a). It is a plot of comparing the experimental and theoretical values of $1/q_l$, $1/q$, and $2\pi/q'$. All of the data included in the graph was taken at around room temperature. Lines 1, 3, and 6 are the experimental data, while lines 2, 4, and 5 are the theory.

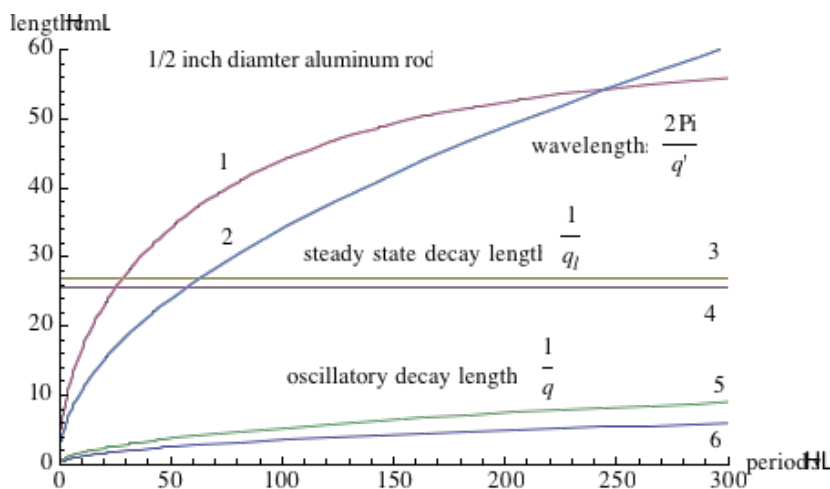


Figure (6b): Experimental vs. Theoretical Data

Although we did not have time to collect enough data to study the thin aluminum rod, I have included Figure (6c), which is theoretical data for a thin rod. Comparing figure s (6b) and (6c) shows that as the diameter decreases, the

⁷ While fitting the data from Period = 150s, I noticed that one data point for a particular x_i was far away from the rest of the data, so I excluded data from that thermistor.

wavelength increases, the steady state decay length decreases, and the oscillatory decay length decreases slightly.

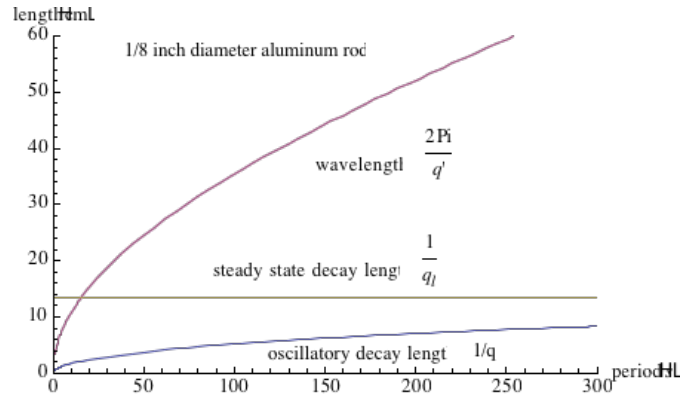


Figure (6c): Theoretical Data

Using our experimental values for $q(\omega)$ and $q'(\omega)$, we were able to solve for the constants κ and ν . From Equation (19), it is obvious that the κ can be found by graphing $q q'(\omega)$ versus ω . If m is the slope of the resulting graph, then $\kappa = 1 / (2m)$. Figure (7) shows the graph of $q q'(\omega)$ versus ω . Fitting the line to a slope gives a value for κ of $9.20 \times 10^{-5} \text{ m}^2\text{s}^{-1}$. When graphing $q q'(\omega)$ versus ω , the data point for $\tau=300\text{s}$ was again thrown away due to its large error.

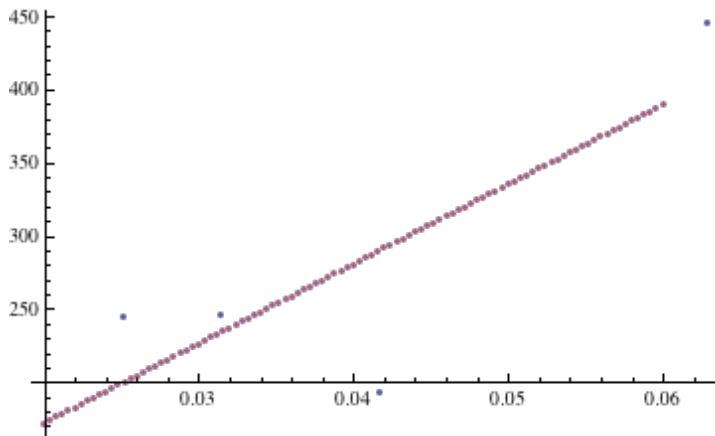


Figure (7): $q q' (\text{m}^2)$ vs. ω

I compared the experimental value of κ to its theoretical values, which is calculated by plugging the constants of the setup into Equation (12):

$$(25) \quad \begin{aligned} \kappa &= k / (c\rho) \\ &= 9.05 \times 10^{-5} \text{ m}^2\text{s}^{-1} \end{aligned}$$

The percent error between the measured value for κ , $(9.20 \pm 0.01) \times 10^{-5} \text{ m}^2\text{s}^{-1}$ and the theoretical value, $9.05 \times 10^{-5} \text{ m}^2\text{s}^{-1}$, is $(1.63 \pm 0.01)\%$, which implies that the data and analysis are relatively accurate.

In a similar manner, we can extract the value of ν from a graph of q' / q . When this expression is simplified, κ cancels out of the equation, allowing us to extract the value of ν by fitting a model to the graph of q' / q versus frequency. The model that we used is given in Equation (26), and Figure (8) shows the graph of q' / q versus frequency. The graph contains both a curve of the measured data and one of the theoretical data (“theoretical” data was obtained by solving $q' / q(\omega)$ for different frequencies, based on the constants of our setup). Again, I left out the same outlier that was not included in the graph of $q q'$, because adding in the point substantially increased the already large error in the measured value for ν .

$$(26) \quad q' / q = [-\nu + \sqrt{\nu^2 + \omega^2}] / \omega$$

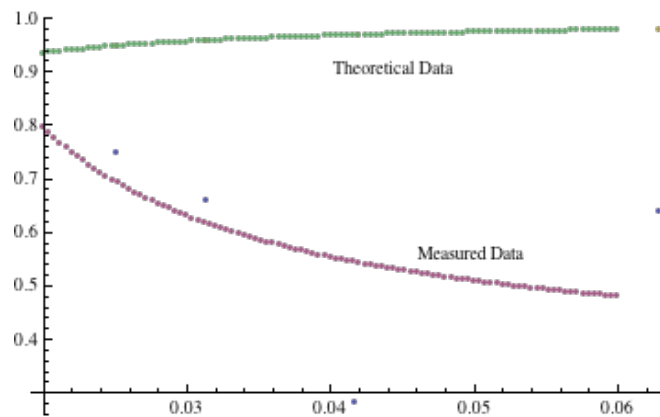


Figure (8): q' / q verses ω

Fitting the data to the model of Equation (26), gives approximately $\nu = 7.62 \times 10^{-3} \text{ s}^{-1}$, but it is clear from the Figure (8) that the model is not the best fit for the data, and that the measured data does not match well with the theoretical data.

$$(27) \quad \begin{aligned} \nu &= hP / (Ac\rho) \\ &= 1.37 \times 10^{-3} \text{ s}^{-1} \end{aligned}$$

We can also calculate the theoretical value of ν by plugging the constants of the setup into Equation (27) (originally Equation (15)). By doing so, we find that ν is equal to $1.37 \times 10^{-3} \text{ s}^{-1}$. Thus, the error in our measured data for ν is significantly larger than the error in κ . Although our experimental value is within the correct order of magnitude, the percent error is very large.

There are, however, several possible reasons for the large amount of error. One significant source of error is in the assumption that h is a true constant. In reality, however, h can vary greatly from situation to situation, and Newton’s Law of Cooling is only an approximation (rather than an absolute physical law). The value of h is dependent on the temperature difference between an object and its surroundings. The fact that the temperature varies along the length of the rod and

that our data was taken over a period of several weeks (in different room temperatures) could have affected our data. While the assumption that h is a perfect constant could have contributed to the error, Newton's Law of Cooling is a reasonable assumption to make for our setup, and the variability of h should not create such a large error.

Another possible source of error is in our LabVIEW code. At the peak and trough of each oscillation, the temperature was not entirely oscillatory. While this alone should not have such a significant affect on our data, it is still an error that should be kept in mind.

Another, more accurate way, to calculate ν is in terms of q_l and κ , by using Equation (28) in conjunction with the experimental values for q_l and κ .

$$(28) \quad q_l = \sqrt{(\nu/\kappa)}$$

Equation (28) gives $\nu = (1.26 \pm 1.01) \times 10^{-3} \text{ s}^{-1}$. There is now only an 8.1% error for ν , which suggests that the large error in the ν found by plotting q'/q versus frequency is probably due to a problem with the modeling on Mathematica, rather than an error in our data or theory.

After finding q , q' , κ , and ν , we were finally able to solve for surface conductance h and the thermal conductivity k for aluminum, using Equations (12) and (13). Using the better value for ν , we find that $k = 224 \text{ Wm}^{-1}\text{K}^{-1} \pm 1 \text{ Wm}^{-1}\text{K}^{-1}$ and $h = 9.19 \text{ Wm}^{-2}\text{K}^{-1} \pm 0.01 \text{ Wm}^{-2}\text{K}^{-1}$. According to the published data for aluminum, $k = 220 \text{ Wm}^{-1}\text{K}^{-1}$ and $h = 10 \text{ Wm}^{-2}\text{K}^{-1}$, meaning that our percent error for k and h are only 1.63% and 8.13%, respectively. Thus, we can conclude that although we encountered errors in some of our data analysis, our data is most likely fairly accurate and in agreement with our theory.

Lastly, now that we know h and k , we can easily solve for the steady state heat flux from the fin, using Equation (9b) and the constants of our setup. We will solve for the heat flux when $T_0 = 40^\circ\text{C}$, because this was our most accurate data point.

$$(29) \quad q = (h P k / A)^{1/2} (T_0 - T_\infty)$$

Using Equation (29), we find that the steady state heat flux is equal to approximately 10.8 kW m^{-2} . While there is no published data for the heat flux in our particular aluminum rod, we can also solve Equation (29) by using the published, rather than experimental, values for h and k . This gives $q = 11.1 \text{ kW m}^{-2}$, with a corresponding 2.7 percent error.

Conclusion

In conclusion, in our analysis of the heat flow through an aluminum rod, we calculated, at around room temperature, that $q_l = 3.70 \text{ m}^{-1}$; $q(\kappa, \nu, \omega) = q(9.20 \times 10^{-5}$

m^2s^{-1} , $1.26 \times 10^{-3} \text{ s}^{-1}$, ω); and $q'(\kappa, \nu, \omega) = q'(9.20 \times 10^{-5} \text{ m}^2\text{s}^{-1}, 1.26 \times 10^{-3} \text{ s}^{-1}, \omega)$. This in turn allowed us to calculate $h = 9.19 \text{ Wm}^{-2}\text{K}^{-1}$ and $k = 224 \text{ Wm}^{-1}\text{K}^{-1}$.

We saw that although some of our models and data had large errors, overall, our experimental data was consistent with our theoretical data. We can therefore assume that there were no major errors in our theory or data.

Lastly, the heat flow through a cylindrical aluminum rod has many applications. One example is the rods in aluminum "pin fins." Arranging a fixed mass of aluminum as a flat base with many round pins jutting out of it creates a pin fin. Because of the round geometry of the pins (large surface area per volume), their omnidirectional arrangement (turbulent air flow between pins), and their conductive material (copper or aluminum), pin fins are conducive to cooling. Pin fins are chosen to cool chips, because they are effective heat sinks and can dissipate large amounts of heat in small volumes. With the help of a small fan, a pin fin can dissipate up to 185 W in a volume of less than 6.3 in^3 . A typical pin fin might have cross sectional base area of 1 in^2 , a total height of 0.5 in., and be able to cool 2 W within a base-temperature rise of 40°C .

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